ON-LINE CHAIN PARTITIONING OF POSETS

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An on-line chain partitioning algorithm receives the points of the poset from some externally determined list. Being presented with a new point the algorithm learns the comparability status of this new point to all previously presented ones. As each point is received, the algorithm assigns this new point to a chain in an irrevocable manner and this assignment is made without knowledge of future points. The value of the on-line chain partitioning problem, $\mathrm{val}\,(w)$, is the least integer k, such that there is an on-line algorithm that never uses more than k chains on posets of width at most w.

Kierstead [2] presented an algorithm using $(5^w - 1)/4$ chains to cover each poset of width w. On the other hand an argument of Endre Szemerédi (presented in [3]) proves the best known asymptotic lower bound $\binom{w+1}{2} \leqslant \mathrm{val}(w)$. Kierstead [2] presented also a lower bound $4w - 3 \leqslant \mathrm{val}\,(w)$ which is better than $\binom{w+1}{2}$ for the first few values $w = 2, 3, 4, 5$. In particular, this lower bound together with an algorithm of Felsner [1] gives the precise value for $\mathrm{val}\,(2) = 5$. The research carried out up to now puts $\mathrm{val}\,(3)$ between $9 = 4 \cdot 3 - 3$ and $31 = (5^3 - 1)/4$. The aim of speach is to present a better upper bound $\mathrm{val}\,(3) \leqslant 16$.

References

[1] Stefan Felsner, *On-line chain partitions of orders*, Theoret. Comput. Sci. **175** (1997), no. 2, 283–292.

[2] Henry A. Kierstead, *An effective version of Dilworth's theorem*, Trans. Amer. Math. Soc. **268** (1981), no. 1, 63–77.

[3] ———, *Recursive ordered sets*, Combinatorics and ordered sets (Arcata, Calif., 1985), Contemp. Math., vol. 57, Amer. Math. Soc., Providence, RI, 1986, pp. 75–102.

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