On-line dimension of semi-orders

Kamil Kloch

We analyze the on-line dimension semi-orders as a two-person game between Algorithm and Spoiler. The game is played in rounds. Spoiler presents an on-line order, one point at a time. Algorithm maintains its realizer, i.e., an embedding into multi-dimensional grid $\mathbb{Q}^d$, or equivalently, d linear extensions arising from projecting the embedding onto d coordinates. Algorithm may not change the ordering of the previously introduced elements in the existing linear extensions. The performance of Algorithm is measured by comparing the number of linear extensions used against the off-line width of the presented poset.
Several variants of the game are being considered. For up-growing semi-orders, we show that the value of the game is exactly w. In the general case we provide an upper bound of $2w$. Moreover, we consider the variant when the order presented by Spoiler is given together with its representation by pairwise non-containing intervals of arbitrary length. In this case we prove a matching lower and upper bound of 4 independently of w.

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