Regular Clustering in UDG in Constant Time

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The Unit Disc Graphs (UDG) is the natural theoretical model to study wireless ad hoc networks (for example cell phones networks, wireless sensor networks etc.). The set of vertices of UDG is a set of points on the plane and two vertices of UDG, v and w, are connected by an edge in UDG if and only if they are at the distance at most one in Euklidian norm ($\left\| v,w\right\| \leq 1$).

One of the fundamental problems concerning this class of graphs is finding a division into clusters which have appropriate number of verices. For example, division into clusters with at least $\dd$ vertices and of size bounded from above by some function $f(\dd)$.

Such construction is useful to simplify routing protocols and is helpful in solving problems in distributed model in which at least $\dd$ parallel procesors are needed. Moreover it may be used in data aggregation and processing of clusters where analyzing more than $f(\dd)$ vertices is very time or memory consuming.

In my considerations I will assume that every node is equipped with the Global Positioning System (GPS), or knows its position on the plane by other sources. In addition, to simplify arguments, I will assume that local clocks of vertices of UDG can be synchronized i.e., assume that we perform computations in rounds (synchronous model).

In this computation model I will present algorithm which, given as an input parameter $\dd$ and a connected Unit Disc Graph G, finds a decomposition of G into the disjoint connected subgraphs $H_1,H_2, \ldots ,H_k$ such that for all ${1\leq i \leq k}$ the inequality $\dd \leq \left|V(H_i)\right| \leq 5\dd +1$ holds. This algorithm is deterministic and work in $O(\dd^3)$ number of synchronous rounds using only short messages. I will also prove that the upper bound $5\dd +1$ is the best possible.