Almost resolvable k-cycle systems

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A {\it k-cycle system} of order n is a pair $(X,{\cal C})$ where ${\cal C}$ is a collection of edge disjoint k-cycles which partition the edge set of the complete undirected graph K_n with $V(K_n)=X$. A k-cycle system $(X,{\cal C})$ is said to be {\it resolvable} if the cycles belonging to ${\cal C}$ can be partitioned into parallel classes.

If $(X,{\cal C})$ is a k-cycle system of order n and k does not divide n then we cannot
have a parallel class of k-cycles. The closest we can come to a parallel class is a collection of $(n - 1)/k$ vertex disjoint k-cycles; any such collection is called an {\it almost parallel class}. The maximum possible number of edge disjoint almost parallel classes in
a k-cycle system of order n is $(n - 1)/2$ in which case a half parallel class
containing $(n - 1)/2k$ vertex disjoint k-cycles is left over. A k-cycle system
of order n whose k-cycles can be partitioned into $(n - 1)/2$ almost parallel classes and a half parallel class is said to be {\it almost resolvable} and is denoted by k-ARCS(n).

The existence of 3-ARCSs was settled in 1993 by H. Hanani. Moreover, quite recently I. Dejter, C. Lindner C. Rodger and M. Meszka proved the existence of 4-ARCSs. Complete solutions for $k=6$ and $k=10$ as well as a complete solution with one possible exception for $k=14$ will be presented