Evaluating monotone Boolean functions and game trees in the priced information model

Martin Milanic

We study the fundamental problem of evaluating a function by sequentially selecting a subset of variables whose values uniquely identify the function's value. We address the variant of the function evaluation problem, introduced by Charikar et al. (2002), where different variables can incur different reading costs and competitive analysis is employed to measure the performance of an evaluation algorithm.

Based on a linear programming approach, Cicalese and Laber (2008) developed a framework for the design of algorithms for evaluating functions. This framework motivates interesting combinatorial problems. In the case of a monotone Boolean function f, the relevant question is the following: Given a Sperner family A, give good upper bounds on the value of a minimal fractional hitting set of the union of A and the set B of all its minimal transversals. The elements of A correspond to the minimal sets of variables of f whose values can be fixed in a way such that the restricted function becomes constantly equal to 1. Similarly, the elements of B correspond to the minimal "proofs" for f=0. It has been shown by Cicalese and Laber (2008) that there is a fractional hitting set of C= A \cup B of value smaller than or equal to the maximum cardinality of a set in C. In turn, this result allowed for the determination of the tight extremal competitive ratio for monotone Boolean functions. Similar questions arise in the design of algorithms for evaluating game trees. While it is an open question whether a small-valued fractional hitting set of the above-mentioned family C can be computed in polynomial time (given A), we show that this is the case for the family of the minimal sets of variables that can prove a lower or an upper bound for the value of T', where T' is a restriction of a game tree. In particular, this allows us to obtain the tight extremal competitive ratio for game trees, and shows that for game trees, optimal competitiveness can be achieved by a polynomial-time algorithm. We also extend the framework of Cicalese and Laber to the case where the cost of querying a variable can depend on the variable's value, and determine the tight extremal competitive ratio for monotone Boolean functions and game trees in this model.

The talk is based on joint work with Ferdinando Cicalese.