Lucky labelings of graphs

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Abstract. Suppose the vertices of a graph G were labeled arbitrarily by positive integers, and let $S(v)$ denote the sum of labels over all neighbors of vertex v. A labeling is *lucky* if the function S is a proper coloring of G, that is, if we have $S(u) \neq S(v)$ whenever u and v are adjacent. The least integer k for which a graph G has a lucky labeling from the set $\{1, 2, \dots, k\}$ is the *lucky number* of G, denoted by $\eta(G)$.

Using algebraic methods we prove that $\eta(G) \leq k + 1$ for every bipartite graph G whose edges can be oriented so that the maximum out-degree of a vertex is at most k. In particular, we get that $\eta(T) \leq 2$ for every tree T, and $\eta(G) \leq 3$ for every bipartite planar graph G. By another technique we get a bound for the lucky number in terms of the *acyclic chromatic number*. This gives in particular that $\eta(G) \leq 100\,280\,245\,065$ for every planar graph G. Nevertheless we offer a provocative conjecture that $\eta(G) \leq \chi(G)$ for every graph G.